Can the lottery paradox be solved by identifying epistemic justification with epistemic permissibility?

Benjamin Kiesewetter

Abstract: Thomas Kroedel argues that the lottery paradox can be solved by identifying epistemic justification with epistemic permissibility rather than epistemic obligation. Since permissions generally do not agglomerate, it seems that we can be permitted to believe of each lottery ticket that it will lose without being permitted to believe that all tickets will lose. In this article, I present two objections to this solution. First, even if justification itself amounts to no more than epistemic permissibility, the lottery paradox re-emerges at the level of epistemic obligations unless Kroedel adopts an extremely permissive view about suspension of belief that is at odds with our practice of epistemic criticism. Second, even if there were no positive epistemic obligations to believe lottery propositions, Kroedel's solution fails because justification agglomerates independently of whether justification amounts to permissibility or obligation.

Keywords: Lottery paradox, epistemic obligation, justification agglomeration, epistemic permissibility, Thomas Kroedel.

1. The lottery paradox and Kroedel's permissibility solution

Thomas Kroedel has recently argued for a novel solution to Henry E. Kyburg's famous lottery paradox.[1] On a common construal, the paradox occurs if we apply two plausible assumptions about epistemic justification to the case of an agent A who knows that he is confronted with a fair lottery with a large number of tickets, one and only one of which will win. The first assumption is that we can justifiably believe what is very likely, given our evidence:

[1] Kroedel (2012); Kyburg (1961, 197). Recent discussions of Kroedel's permissibility solution include Eder (2015), Huber (2014), Littlejohn (2012; 2013) as well as Kroedel (2013a; 2013b). Neither of these contributions addresses the objections that I raise in this paper.

The probability claim: If A's evidence makes p exceedingly likely, then A has justification for believing p.[2]

Let us call a proposition of the form 'lottery ticket number *x* will lose' a *lottery proposition.* Depending on the number of tickets involved in the lottery, the probability claim entails that for each lottery proposition, A has justification to believe it. By varying the number of tickets, we can generate this result for any degree of probability that we deem sufficient for justification, as long as it is below 1.

The second assumption is that we have justification for believing a proposition if that proposition is a conjunction of two other propositions each of which we can justifiably believe:

The conjunction claim: If A has justification for believing p*,* and A has justification for believing q, then A has justification for believing p&q.[3]

The well-known problem is that these two initially plausible assumptions together entail the paradoxical conclusion that A has justification for believing that *all* lottery tickets will lose. To see this, let 'J' be the operator for justification, let 'B' be the operator for belief,

[2] Three clarificatory remarks on the relevant notions of 'justification' and 'belief': First, like Kroedel, I am primarily concerned with *epistemic* justification. Except when I explicitly address other possible forms of justification for doxastic states, I use "justification" as short for "epistemic justification". Second, like Kroedel, I am concerned with what is sometimes called "propositional justification" (in contrast to "doxastic justification" which is concerned with the epistemic status of a particular belief token). Throughout this paper, I use "A can justifiably believe p" and "A has justification for believing p" equivalently in order to refer to this kind of justification, where "having justification" in this paper always means having *sufficient* justification, not merely having partial justification. Third, again like Kroedel, I am concerned only with beliefs *simpliciter,* not with the quantitative notion of belief that allows for degrees. The lottery paradox is often discussed in the context of what Foley (1992, 111) has dubbed the "Lockean Thesis", according to which the standards of justification for belief *simpliciter* can be derived from the standards of justification for the degree of confidence that amounts to belief *simpliciter*. Despite what is sometimes asserted, however, the Lockean Thesis is not an essential premise of the lottery paradox, but only one possible motivation for the probability claim. It is perfectly consistent to deny the assumption (entailed by the Lockean Thesis) that the ordinary notion of belief *simpliciter* can be explained in terms of a quantitative notion of belief – as, for example, Skorupski (2010, 51) does – while maintaining the probability claim with respect to belief *simpliciter*.

[3] The conjunction claim also follows from the more general *closure principle,* according to which we have justification for believing a proposition if it logically follows from other propositions each of which we can justifiably believe. I shall focus on the conjunction claim, however, since no stronger claim is needed to generate the paradox.

let 'l_x' be the proposition 'lottery ticket number *x* will lose', and let 'n' be the number of tickets that the lottery contains. The argument can then be represented as follows:

(1) $JBl_1\&\ldots\&JBl_n$
(2) $(JBp\&JBq)\rightarrow JB(p\&q)$
(3) $JB(l_1\&\ldots\&l_n)$

(1) states that for each lottery ticket, A has justification to believe that this ticket will lose, which is entailed by the probability claim. Through iterated applications of the conjunction claim (2), we reach the conclusion (3) that A has justification to believe that all tickets will lose. This conclusion is clearly false: we assumed that A *knows* that at least one ticket will win; the probability of the proposition that all tickets will lose is thus 0, and A cannot have justification for believing it. Kyburg concluded from this that justification is not as closely related to logic as it might have seemed and rejected the conjunction claim (2).[4] While some epistemologists have followed him, others again resisted this conclusion and rejected the probability claim instead, thereby escaping the paradox by denying (1).[5]

How does Kroedel's permissibility solution relate to this dialectic? Although his own presentation of the paradox conceals this fact, Kroedel actually sides with Kyburg in giving up the conjunction claim, as it is usually understood. In contrast to Kyburg, however, Kroedel thinks that there is an alternative interpretation of the conjunction claim that can be maintained in the light of the lottery paradox even if we hold on to the probability claim. According to this alternative interpretation, the justification operator in the antecedent of the conjunction claim takes wide scope over 'believing p and believing q':

The wide-scope conjunction claim: If A has justification for believing p and believing q, then A has justification for believing p&q. [$J(Bp\&Bq)\rightarrow JB(p\&q)$]

---

[4] See also Kyburg (1970).

[5] See e.g. Foley (1979) and Klein (1985) for the rejection of the conjunction claim, and Ryan (1996) and Nelkin (2000) for the rejection of the probability claim.

If we replace (2) with this wide-scope version of the conjunction claim, the paradoxical conclusion (3) does not follow anymore. This solution requires, however, that the following principle is false:

Justification agglomeration: If A has justification for believing p, and A has justification for believing q, then A has justification for believing p and believing q. [(JBp&JBq)→J(Bp&Bq)]

This is because Kroedel's solution is supposed to preserve the probability claim and its implication that for each lottery proposition, A has justification to believe it (1). If justification agglomerates, however, it follows from (1) that A has justification for having all lottery beliefs together and we can run the following argument to the paradoxical conclusion:

$(1)_{WS}$ $J(Bl_1\&\ldots\&Bl_n)$, *from (1) by justification agglomeration*
$(2)_{WS}$ $J(Bp\&Bq)\rightarrow JB(p\&q)$, *wide-scope conjunction claim*
(3) $JB(l_1\&\ldots\&l_n)$

Kroedel's attempt to rescue a version of the conjunction claim while holding on to the probability claim thus depends essentially on his rejection of *justification agglomeration*. His argument is that since "justification is a species of permissibility"[6], and "permissibility does not agglomerate"[7], justification likewise fails to agglomerate. If it works, his solution would be a significant achievement, since we could maintain the intuitive connection between justification and probability while holding on to at least a version of the conjunction claim.[8]

---

[6] Kroedel (2012, 57). Note that Kroedel does not actually explicitly endorse the claim that justification is a species of permissibility; strictly speaking, he argues only for the conditional claim that *if* justification is a species of permissibility, then this provides a solution to the lottery paradox.

[7] Kroedel (2012, 59).

[8] Kroedel's presentation of both the paradox and his solution differs from mine in some respects. In his original article (Kroedel 2012), he does not even mention what I take to be the standard interpretation of

In this article, I present and discuss two objections to Kroedel's solution. The first objection grants Kroedel's assumption that justification by itself amounts to no more than an epistemic permission, but argues that it is nonetheless plausible to assume that there are epistemic obligations to believe highly probable propositions under certain conditions that a lottery case might satisfy. It follows that the lottery paradox can be restated in terms of such obligations rather than in terms of justification (§2). The second objection is that Kroedel's solution fails even if there are no obligations to believe lottery propositions, because it requires that justification fails to agglomerate. I argue that *justification agglomeration* is plausible independently of whether justification amounts to no more than epistemic permissibility, and that denying this principle creates a dilemma (§3).

2. First objection: There are epistemic obligations

As we have seen, Kroedel's solution to the lottery paradox is based on the assumption that a person's having justification for a belief is to be identified not with this person's being *obligated* or *required,* but with this person's being *permitted* to have this belief.[9] This is a tenable view, and I shall grant it for the sake of the argument. As Gilbert Harman in particular has emphasised, there is no point in cluttering one's mind with an abundance of trivial or otherwise uninteresting beliefs, even if we could justifiably have them.[10] And as Mark Nelson has recently argued, a duty to believe every proposition that our evidence justifies would amount to a duty that we could not possibly satisfy, since our evidence always justifies an infinite number of possible beliefs.[11] Plausibly, then, we are not obligated to believe everything we are in a position to believe justifiably.

---

the conjunction claim (2), but simply assumes the wide-scope version $(2)_{WS}$ as the relevant claim at issue. Disregarding (2), he then argues that the paradoxical conclusion follows only if the first premise is interpreted as the wide-scope claim $(1)_{WS}$ rather than the narrow-scope claim (1). Kroedel thus seems to take the lottery paradox to consist in the fact that $(1)_{WS}$ and $(2)_{WS}$ entail (3), and his solution to consist in substituting (1) for $(1)_{WS}$. In my view, he thereby overlooks the central paradox, which consists in the fact that (1) and (2) entail (3). In a later discussion (Kroedel 2013a, §2), he acknowledges this version of the paradox and explicitly states that he rejects not only $(1)_{WS}$ but also (2). Ultimately, these are matters of presentation only. Whether the central paradox is that (3) is entailed by (1) and (2) or that it is entailed by $(1)_{WS}$ and $(2)_{WS}$, in either case Kroedel accepts both (1) and $(2)_{WS}$, while rejecting both $(1)_{WS}$ and (2), and in either case this commits him to denying *justification agglomeration.*

[9] In this article, I use the terms 'obligation', 'requirement' and 'duty' interchangeably in order to refer to the epistemic 'ought'. No moral connotations are intended by the use of any of these terms.

[10] See Harman (1986, 12–15).

[11] See Nelson (2010, esp. 96–98).

However, the truth of this claim does not suffice for Kroedel's solution to succeed. That justification does not necessarily amount to epistemic obligation does not entail that justification does not give rise to epistemic obligation *under certain conditions that a lottery case might satisfy.* And if there are such conditions under which justification gives rise to epistemic obligation, then the lottery paradox can be restated in terms of such obligations, no matter whether justification as such amounts to permissibility or obligation. To illustrate this, consider:

Bridge pattern: If A has justification for believing p, and condition C obtains, then A ought to believe p.

Different views about how condition C needs to be understood for *Bridge pattern* to come out as true are possible. Drawing on Gilbert Harman, we may say that C is the condition that A is interested in whether p is the case.[12] Drawing on Robert Nozick, we may say that C is the condition that A's expected utility of believing p is higher (or at least not lower) than the expected utility of having no belief about whether p is the case.[13] My own view is that C is the condition that A pays sufficient attention to the question of whether p is the case.[14] But my aim here is not to defend a particular view about condition C. I mention these different accounts only to illustrate that there are various ways to avoid the implausible implications of a view that identifies justification with epistemic obligation while still holding on to the intuitively plausible claim that we are sometimes required, and not merely permitted, to have epistemically justified beliefs.

Before I will say a bit more in defence of this claim, let me briefly explain the problem that it poses for Kroedel's permissibility solution. Suppose that one of the mentioned views about condition C is correct. We may then stipulate a lottery case in

[12] Cf. Harman (1986, 55).

[13] Cf. Nozick (1993, 86).

[14] See Kiesewetter (2013, ch. 7.7). One argument in favour of this latter view is that it makes good sense of how the assumption that sufficient evidence for p is not by itself enough to ground an obligation to believe p fits together with Richard Moran's famous observation that the first-personal deliberative question of whether to believe p is transparent on the theoretical question of whether p is the case (cf. Moran 1988, 143). My account of condition C explains this, because it entails that *once one considers this question in deliberation*, sufficient evidence for p *does* suffice for an obligation to believe p.

which condition C is satisfied for every lottery proposition. That is, we may stipulate a lottery case in which A is interested in the truth of each of the lottery propositions, or A's expected utility of believing each lottery proposition is higher than the expected utility of suspending judgement, or A pays sufficient attention to the truth of each lottery proposition. Our bridge principle will then entail, together with the probability claim, that for each lottery proposition, A ought to believe it.[15] And since epistemic obligations agglomerate (as Kroedel agrees), and epistemic obligation implies epistemic justification, we will not be able to avoid the paradoxical conclusion that A has justification to believe that all tickets will lose unless we give up the conjunction claim in the wide scope reading that Kroedel wants to preserve as well. Taking 'O' as the operator for epistemic obligation, this argument can be represented as follows:

(4) $OBl_1\&\ldots\&OBl_n$

(5) $O(Bl_1\&\ldots\&Bl_n)$, from 4 by *obligation agglomeration* $[(OBp\&OBq)\rightarrow O(Bp\&Bq)]$

(6) $J(Bl_1\&\ldots\&Bl_n)$, from 5 by *obligation implies justification* $[O(Bp)\rightarrow J(Bp)]$

(7) $JB(l_1\&\ldots\&l_n)$, from 6 by *wide-scope conjunction claim* $[J(Bp\&Bq)\rightarrow JB(p\&q)]$

Kroedel thus needs to reject any instance of the bridge pattern that, together with the probability claim, entails (4).

Kroedel is committed to the existence of epistemic obligations himself: he maintains that in a lottery case, we are not permitted to believe that all tickets will lose, and he holds that this is equivalent to saying that we are obligated not to believe that all tickets will lose.[16] This, however, is a *negative* epistemic obligation, while the bridge pattern generates *positive* epistemic obligations. To escape the problem, Kroedel may thus want to follow Nelson (to whom he refers in a note) in denying the existence of any *positive* epistemic obligations.

[15] It is no response to say that since agents like us are psychologically incapable of attending to all lottery propositions at the same time, there is no time at which A is obligated to have each lottery belief. Apart from the fact that this reply presupposes an unnecessarily restrictive interpretation of the attention condition, it ignores that we may simply stipulate an agent who has this capacity, and that we still want to deny that such an agent has justification for believing that all tickets will lose.

[16] See Kroedel (2012, 58) for the first of these claims, and Kroedel (2013a, 108) for the second.

Digression on Nelson

Nelson's view does not actually support Kroedel's solution, however. This is because, despite first appearances, Nelson does not argue that there is no true instantiation of the bridge pattern. Rather, Nelson argues that a true instantiation of the bridge pattern must mention non-epistemic considerations in C, and that *for this reason,* obligations to believe are never really *epistemic* obligations:

> My thesis is that there is nothing we positively ought to believe simply in virtue of our epistemic circumstances, and nothing that we 'ought' to believe at all, except given some further interest, desire, duty, or such like.[17]

I disagree with Nelson on two points. For one, as should be clear from what I have said above, I do not think that condition C must involve a reference to non-epistemic conditions, and I do not think that Nelson's argument shows anything like that. Nelson argues that the quality of an agent's evidence alone never suffices for an obligation to believe, but it does not follow that such obligations must be conditional on "some further interest, desire, duty, or such like". The relevant condition may, for example, be that A pays attention to the relevant proposition, and this is arguably part of A's "epistemic circumstances". For another, even if Nelson is right to say that C must involve a non-epistemic condition, it seems to me that the obligations resulting from the bridge pattern may still legitimately be called *epistemic* obligations (or epistemic duties, or epistemic requirements) as long as those obligations are not in any way based on non-epistemic reasons for belief. Nelson does not claim that obligations to believe are based on non-epistemic reasons for belief, however. His view is rather that sufficient epistemic reasons lead to obligations only if practical *background conditions* are satisfied. It seems to me perfectly fine to call such obligations epistemic obligations nonetheless. By way of an analogy, it seems to me a perfectly intelligible and consistent view to maintain that there

[17] Nelson (2010, 92).

are prudential requirements to act in accordance with weighty prudential reasons, but only if certain moral background conditions are satisfied.

Most importantly, however, Nelson's view would not help Kroedel's permissibility solution *even if* we accepted his position that there are no epistemic obligations. For as long as we maintain that some instance of the bridge pattern provides obligations to believe – and Nelson does not deny this – the lottery paradox can be restated in terms of such doxastic obligations, no matter whether these are to be called epistemic obligations or not.

Doxastic permissivism and epistemic criticism

It thus seems that Kroedel must go further than Nelson and deny that there are any positive doxastic requirements that are generated along the lines of the bridge pattern at all, at least when the probability of the relevant proposition is below 1. According to such a view, suspension of belief is almost always permitted:

Doxastic permissivism: No matter how carefully we attend to the question of whether p, and no matter how important that question is from a practical standpoint, as long as the probability of p is less than 1, we are always permitted to refrain from believing p.

This extremely permissive view is not only *prima facie* implausible, it would undermine the practice of epistemic criticism if it were true. We criticise persons not only for believing against their evidence, but at least sometimes also for not believing in accordance with their evidence. Such criticism presupposes that there are obligations, and not just permissions, to have beliefs. Just as blame in general involves the assumption that an agent has failed to conform to a requirement and not merely omitted to make use of a permission, epistemic blame or criticism involves the assumption that the agent has failed to conform to an epistemic requirement, not merely omitted to make use of an epistemic permission.[18]

[18] Kroedel (2013a, 107–8) himself accepts that epistemic blameworthiness entails impermissibility and thus violation of an epistemic obligation. For an argument to the more general conclusion that a person is

Consider Smith and Jones, two climate scientists who are both tied to political organisations that deny climate change. Both have very strong evidence for climate change, even though there is a residual probability that the data are misleading. Both are interested in the question of whether climate change is happening and pay sufficient attention to it. While Smith believes that climate change is not happening, Jones suspends judgement on that question and does not form a belief about climate change at all. It seems to me that Smith and Jones are both rationally criticisable for their doxastic states. Jones does not escape such criticism merely by suspending judgment.[19] This, however, can be so only if there are obligations not only to abstain from beliefs that are unlikely to be true, but also to have beliefs that are likely to be true (under certain conditions).[20]

Can Kroedel avoid doxastic permissivism?

On behalf of Kroedel, it might now be suggested that there may be an account of the relevant condition C of the bridge pattern that excludes the possibility of epistemic obligations to believe lottery propositions, while at the same time allowing the existence of obligations to believe other propositions that are less than absolutely certain. It is unclear, however, whether an account of such a condition can be given without introducing extremely *ad hoc* assumptions about the content of propositions that we may be required to believe (such as e.g. the assumption that a belief can be required only if it is not about a lottery ticket). To see this, let us briefly consider some attempts to provide a condition that rules out lottery propositions on grounds of their relevant structural features. At first sight, it might seem that Kroedel could put forward a view according to which A has an obligation to believe a proposition p that she can justifiably believe if

criticisable for omitting to F only if she has decisive reasons, or ought all things considered, to F, see Kiesewetter (2013, ch. 2).

[19] This also counts against Nozick's view. We may assume that Jones' expected utility of suspending judgement is higher than that of forming a belief about climate change, but he seems rationally criticisable for suspending judgement nonetheless.

[20] Some authors deny this on the assumption that one cannot justifiably believe p if one knows that one does not know p (cf. e.g. Littlejohn 2012, 512). I take it, however, that the point I am making about epistemic criticism also counts against this restrictive assumption. In any case, the response is not available to Kroedel, since it is an essential part of his solution to preserve the initially plausible assumption that for each of the tickets, one can justifiably believe that it will not win.

C1 A attends to p[21] *and* p is consistent with the conjunction of all other propositions that A justifiably believes.

But unless we assume that A has already formed beliefs about lottery propositions (which is not essential for a lottery case), this does not rule out the assumption that Kroedel wishes to avoid, namely that for each lottery proposition, A is obligated to believe it. Alternatively, Kroedel might suggest that A has an obligation to believe a proposition p that she can justifiably believe if

C2 A attends to p *and* p is consistent with the conjunction of all other propositions that A can justifiably believe.

Yet since Kroedel accepts the probability claim, and thus the assumption that for each lottery proposition, A can justifiably believe it, and since A knows how many tickets are involved and that one of them will win, it follows that the conjunction of all propositions that A can justifiably believe is already inconsistent, and so C2 effectively rules out all positive doxastic obligations. C2 might be relaxed by saying that A has an obligation to believe a proposition p that she can justifiably believe if

C3 A attends to p *and* p is consistent with each proposition q that A can justifiably believe.[22]

But this condition seems too weak to exclude obligations to have lottery beliefs. Consider a particular lottery proposition l_x. In order for C3 to exclude an obligation to believe l_x,

---

[21] Some condition like this is still necessary to avoid Harman's and Nelson's points discussed above. The reader is free to replace this part with her favourite theory of how to account for these points.

[22] Alternatively, one might also put forward C3*: A attends to p *and* p is consistent with every conjunction of propositions $p_1\&\ldots\&p_n$ that is such that A can justifiably have the belief-set $\{B(p_1), \ldots, B(p_n)\}$. I do not discuss this condition separately because I take it that on Kroedel's view, C3 and C3* are coextensional. The reason for this is that according to any plausible view, if A has justification for believing $(p_1\&\ldots\&p_n)$, then A has justification for having the belief-set $\{B(p_1), \ldots, B(p_n)\}$. This claim, together with the wide-scope conjunction claim, entails the following *set justification principle:* A has justification for having the belief-set $\{B(p_1), \ldots, B(p_n)\}$ if, and only if, A has justification for believing $(p_1\&\ldots\&p_n)$. And on the assumption of this principle, C3* is satisfied if, and only if, C3 is satisfied.

there would need to be a proposition q such that in a lottery case, A can justifiably believe q, and q is inconsistent with l_x. But since l_x is extremely likely, every proposition inconsistent with l_x is extremely unlikely. And so we should not assume that A could justifiably believe it. To illustrate this point, an example that comes to mind for a proposition that is inconsistent with l_x is the conjunction consisting of the proposition that one ticket will win and all lottery propositions but l_x. But since that conjunction is extremely unlikely, we should not assume that A could justifiably believe it. The same will be true for every other proposition that is inconsistent with l_x. For this reason, C3 does not rule out an obligation to believe l_x.

To escape this problem, it seems that Kroedel has to put forward a stronger condition, according to which A has an obligation to believe a proposition p that she can justifiably believe if

C4 A attends to p *and* A has justification to believe p in conjunction with each proposition q that A can justifiably believe.[23]

This condition will indeed exclude requirements to believe lottery propositions. Since A cannot justifiably believe the conjunction of all lottery propositions, there will be some conjunction of lottery propositions $l_1\&\ldots\&l_{max}$, such that A cannot justifiably believe the conjunction $l_1\&\ldots\&l_{max+1}$. And so C4 rules out obligations to believe lottery propositions.

The problem is that C4 rules out all other obligations to believe propositions that are less than absolutely certain as well. Think of a proposition q such that the probability P(q) is *barely sufficient* to render a belief in q justified – any further decrease of P(q) would mean that q is insufficiently likely for justification. Plausibly, for every proposition p that we can justifiably believe, there will be a p-independent proposition q that is barely sufficiently likely among the multitude of propositions that we can justifiably believe (we can easily arrive at such a barely sufficiently likely proposition by conjoining other propositions that are more likely to a conjunction). Now suppose that the probability of p

[23] Alternatively, one might also put forward C4*: A attends to p *and* for every belief-set S that A can justifiably have, the set that results from adding the belief that p to S is one that A can also justifiably have. For the reasons mentioned in the preceding note, on Kroedel's view C4* is satisfied if, and only if, C4 is satisfied.

is below 1. Provided that p and q are independent, it follows that the probability P(p&q) is lower than the probability P(q) and thus insufficient for justification. Thus, on the reasonable assumption that among the propositions that we can justifiably believe is one that is barely sufficiently likely, C4 rules out all requirements on beliefs that are less than absolutely certain.

The upshot of this discussion is that it is far from clear whether a condition can be found that rules out obligations to have lottery beliefs on grounds of considerations concerning their relevant structural features without ruling out all obligations to have beliefs that are less than absolutely certain. This should not surprise us. On the most general level, what generates the lottery paradox is the fact that the probability of a conjunction of two independent propositions that are less than absolutely certain is generally lower than the probability of the conjuncts.[24] This is a general feature of propositions that are less than certain: when such propositions are conjoined in a conjunction, probability gets lost. If one wishes to avoid the implausible conclusion that one can have justification for believing highly unlikely conjunctions, then one either needs to reject obligations to have beliefs that are less than certain, or one needs to reject the claim that one can always justifiably believe the conjunction of all propositions that one is individually obligated to believe. That latter claim is, as the argument from (4) to (7) shows, a commitment of anyone who accepts the wide-scope conjunction claim (it is also a commitment of anyone who accepts the narrow-scope conjunction claim)[25]. And so anyone who like Kroedel wishes to preserve the conjunction claim (in either the narrow- or the wide-scope version) has to deny the existence of obligations to have beliefs that are less than absolutely certain and thus adopt doxastic permissivism.

Let me briefly sum up this section. That justification does not generally amount to epistemic obligation seems plausible, but it does not entail that there are no obligations to have justified beliefs if certain background conditions are satisfied. If there are such obligations, however, then the lottery paradox can be restated in terms of obligation rather

[24] The same fact is also what generates the structurally very similar paradox of the preface, put forward by Makinson (1965).

[25] This is clear from the fact that since epistemic obligation entails epistemic justification, (4) entails (1), and given the narrow-scope conjunction claim [2], (1) entails (3).

than justification on the assumption that the conditions in question are satisfied. To avoid the lottery paradox, or structurally similar paradoxes, Kroedel needs to rule out all positive epistemic obligations to have beliefs that are less than absolutely certain. I have argued that this commitment, which Kroedel shares with those who want to solve the paradox by rejecting the probability claim, is at odds with our practice of epistemic criticism and makes his view implausibly permissive.

3. Second objection: Justification agglomerates

I have argued that even if justification amounts to no more than epistemic permissibility, the lottery paradox re-emerges at the level of epistemic obligations unless Kroedel adopts an implausibly permissive view about suspension of belief. In this section, I shall argue that his solution fails even if we accept such a permissivist view. This is because justification agglomerates even if justification amounts to no more than permissibility, and Kroedel's solution (as we have seen) requires that justification fails to agglomerate.

Let me begin with a logical point. Kroedel's argument against *justification agglomeration* is that justification is a species of permissibility and permissions do not agglomerate. Kroedel is right to say that permissions do not *generally* agglomerate. His example illustrates this point well: one may be permitted to eat this piece of the cake and permitted to eat that piece of the cake without being permitted to eat both of these pieces. This shows that *practical* permissions do not agglomerate, and, moreover, that it is not a conceptual truth that all permissions agglomerate. What it does *not* show, however, is that *epistemic* permissions do not agglomerate. For even if it is not a conceptual truth that permissions agglomerate, and not a substantial normative truth that practical permissions agglomerate, it may still be a substantial normative truth that epistemic permissions agglomerate. Thus, one cannot conclude (as Kroedel seems to) from the fact that practical permissions fail to agglomerate that epistemic permissions fail to agglomerate as well. This conclusion requires additional assumptions about the ethics of *belief*.

This logical point gives rise to a problem for Kroedel's solution, for once we address the relevant normative question directly, it seems actually quite plausible to think that epistemic permissions do indeed agglomerate. It is a truism that an agent can justifiably believe p just in case she has sufficient evidence for believing p.[26] The relevant agglomeration principle thus states no more than that we are always epistemically permitted to have all those beliefs together for which we have sufficient evidence. This strikes me as an intuitively very plausible principle, and, controversial cases like the lottery case aside, it is difficult to imagine a counterexample.

One might argue that the lottery case itself provides reasons to question this principle. That is, one might argue from the probability claim and the wide-scope conjunction claim to denying *justification agglomeration*.[27] But as long as there is no independent reason to question *justification agglomeration*, it is unclear why Kroedel's permissibility solution to the lottery paradox is in any way preferable to the classical solution of Kyburg and others. While the classical solution rejects the conjunction claim in both the standard interpretation and the wide-scope interpretation, Kroedel rejects the conjunction claim in the standard interpretation and *justification agglomeration.* Both of these views then come at the cost of denying principles with *prima facie* plausibility, and it is difficult to see how considerations concerning lottery cases themselves could be able to show why Kroedel's view has any advantage over the classical solution, given that the same cases can also be used to motivate the classical solution.

Denying justification agglomeration creates a dilemma

I have argued that Kroedel lacks a good argument against *justification agglomeration*. I shall now provide an argument in its favour. The argument is that denying this principle creates a dilemma. If one denies *justification agglomeration,* then one is committed to the following possibility: While forming a number of individually justified beliefs, a person reaches the point where she is no longer justified in having all these beliefs together. Since she is not justified, she is not permitted to have these beliefs, and since she is not permitted, she is obligated not to have them. So anyone who rejects *justification agglomeration* is committed to the existence of epistemic obligations not to have certain

[26] Recall that we are concerned with *epistemic* justification only. By 'sufficient evidence' I mean 'evidence sufficient for providing justification for a belief'. What I say here is compatible with radical views according to which evidence is sufficient only if it guarantees truth.

[27] This is, in effect, the argument against *justification agglomeration* that Kroedel submits in his later paper (2013b, 452–53).

sets of beliefs that are such that each belief is individually justified. But how are we to satisfy such an obligation? I shall argue that the opponent of *justification agglomeration* faces a dilemma here. On the first horn of this dilemma, he maintains that it can be fully rational to revise a belief on the basis of the very same evidence on the basis of which one formed the belief in a fully rational way. On the second horn, he accepts that there is no rational way to satisfy that epistemic obligation.

Let us consider the lottery case as an example for an alleged failure of *justification agglomeration.* Kroedel holds that we have justification for believing each lottery proposition l_1, …, and l_n, but we do not have justification for having all of these beliefs together. So if a person forms lottery beliefs, starting from $B(l_1)$, she will at some point acquire a lottery belief $B(l_{max})$, such that if she adds a further lottery belief $B(l_{max+1})$ to her set of lottery beliefs, she is no longer justified in having these beliefs together.[28] Now consider the case of Lotta. Lotta acquires individually justified lottery beliefs, and finally arrives at the belief-set $\{B(l_1), \ldots, B(l_{max+1})\}$ that, according to Kroedel, she is not justified in having. Since Lotta is not justified, and thus not permitted, to have this set of beliefs, she is obligated *not* to have it. But how is she going to satisfy this obligation?

In order to satisfy the obligation not to have the belief-set $\{B(l_1), \ldots, B(l_{max+1})\}$, Lotta needs to revise at least one of her lottery beliefs. But how is she going to do this in a rational way? Plausibly, we can rationally form and revise a belief only on the basis of "object-given" (or "content-related") rather than "state-given" (or "attitude-related") reasons.[29] Object-given reasons for and against beliefs are provided by facts that bear on the truth of the object or the content of the relevant belief, while state-given reasons (if there are any such reasons) are provided by facts concerning the state of having or lacking a belief, for example by considerations concerning the benefits of having or lacking a belief. The standard view is that all reasons for doxastic states are object-given – a view

[28] See also the exchange between Littlejohn (2012; 2013) and Kroedel (2013a; 2013b), esp. Kroedel (2013a, §3). Further below, I assume on behalf of Kroedel that max+1 is reached exactly when the conjunction of l_1, and …, and l_{max+1} is too unlikely to be justifiably believed. But this is not an essential assumption of my argument – as far as this argument is concerned, max+1 might already be reached when the person acquires the belief that l_2.

[29] See Parfit (2001, §2) for the object-given/state-given distinction and Piller (2006) for the content-related/attitude-related distinction.

that has been forcefully argued for by Derek Parfit and Niko Kolodny, among others.[30] But even those who insist on the existence of state-given reasons usually accept that they are irrelevant for the rationality of belief (or, if one thinks that this is different, for the *epistemic* rationality of belief). It is a widely accepted point that insofar as processes of belief-formation and -revision are epistemically rational, they are guided only by evidential considerations (in the broad sense of considerations that bear on the truth of the content of the relevant belief).[31]

This poses a problem for Kroedel. Let t_{max} be the point in time when Lotta has acquired the belief set $\{B(l_1), \ldots, B(l_{max})\}$, and t_{max+1} the point in time when she has acquired the belief-set $\{B(l_1), \ldots, B(l_{max+1})\}$. According to Kroedel, Lotta is required to give up a lottery belief at t_{max+1}, but not at t_{max}. Her evidence, however, does not change between t_{max} and t_{max+1} – all that changes is that Lotta adopts one further lottery belief. But Lotta cannot rationally revise one of her lottery beliefs on the basis of the consideration that she already has too many other lottery beliefs. Considerations concerning the lottery beliefs that Lotta has adopted have nothing to do with the *content* of any of her lottery beliefs; if such considerations provided reasons for or against lottery beliefs, those reasons would be state-given rather than object-given reasons.

Kroedel here faces a dilemma. Either he maintains that Lotta's evidence provides sufficient object-given reasons for revising one of her lottery beliefs, or he accepts that this is not the case. On the first horn, Kroedel is committed to the perplexing claim that a body of evidence that indicates that a proposition p is *extremely likely* provides sufficient reason for *revising* a belief in p. This is doubly implausible. For one, because it is in itself

---

[30] See Parfit (2011, App. A); Kolodny (2005, §3).

[31] Note that this is compatible with the assumption that we can rationally suspend judgment on whether p on the basis of reasons that have nothing to do with the truth of p, e.g. on the basis of the reason that one is tired or distracted. Schroeder (2012, §2) puts forward similar cases in order to show that there can be state-given reasons *against* beliefs (and other attitudes). I agree with those who think that such reasons are better understood as reasons against performing certain *actions,* such as the action of making up one's mind about p, rather than as reasons against the *doxastic state* of believing p (see e.g. Shah and Silverstein 2013). Regardless of this question, however, such cases do not cast doubt on my assumption that we can rationally *revise* a belief that we *already have* only on the basis of object-given reasons. It seems clear that one cannot rationally revise a belief for the reason that one is distracted, for example. (One may rationally revise a belief in p on the basis of the consideration that one has formed the belief under epistemically questionable circumstances such as distraction, but only if one thinks that this casts doubt on the assumption that the evidence that one took oneself to have *for p* was sufficient. Again, this is not an example of a rational belief revision on the basis of reasons that have nothing to do with the truth of p.)

implausible to assume that evidence indicating that there is an extremely small chance that a belief is false provides sufficient reasons to revise that belief. For another, because it is implausible to assume that the very same body of evidence can provide sufficient reasons for both forming *and* revising the *same* belief, and Kroedel is already committed to saying that Lotta's evidence provides sufficient object-given reasons for each lottery belief. So the first horn of the dilemma commits Kroedel to denying the following plausible principle:

Asymmetry principle: If some body of evidence provides a sufficient basis for rationally forming the belief that p, then the same body of evidence does not provide a sufficient basis for rationally revising the belief that p.

On the second horn of the dilemma, Kroedel accepts that Lotta lacks sufficient object-given reasons for revising any of her lottery beliefs, and thus concedes that there is no rational way for Lotta to satisfy her epistemic obligation. Accordingly, Lotta can satisfy her obligation only through some irrational process such as wishful thinking or reasoning on the basis of insufficient reasons or some non-rational process such as forgetting. I take it, however, that the absence of a rational process by which one can satisfy an epistemic obligation undermines the claim that such an obligation exists to begin with. Even though the epistemic 'ought' does not imply voluntary control over doxastic states,[32] it surely implies some sort of 'can' that goes beyond the possibility of irrational or accidental conformity. In any case, it should be clear that a commitment to epistemic obligations that cannot rationally be satisfied would be a significant cost of Kroedel's solution.

The problem generalises for everyone who denies *justification agglomeration.* If justification fails to agglomerate, then it is possible that a number of beliefs are individually but not collectively justified. This means that a person who has formed these beliefs is under an epistemic obligation to give up one of her individually justified beliefs. And this creates a dilemma: either we have to assume that it can be fully rational to revise a belief on the same evidence on which it was rational to form that very belief, or we have

[32] Cf. Chuard and Southwood (2009).

to accept that there are epistemic obligations that cannot rationally be satisfied. Better, then, to stick with *justification agglomeration.*

Objections

In the remainder of this section, I shall discuss two objections to the argument I have given. The first objection is that the first horn of the dilemma can be accepted, because the asymmetry principle is independently questionable. For one, it may be argued that there are *borderline cases* in which the evidence probabilises a proposition to a degree such that one can rationally form as well as revise a belief in that proposition on the basis of that evidence. For another, phenomena that are being discussed under the heading *pragmatic encroachment* seem to show that the standards for knowledge and justification can depend on the pragmatic context,[33] and it seems to follow from this that it can be rational to revise a belief in p on the basis of the same body of evidence on the basis of which one has rationally formed it. To give just one example, it may be argued that it can be rational to revise a belief that the time is such-and-such, which one has rationally formed on the basis of testimonial evidence, if one has learned in the meantime that someone's life depends on what time it is now, even though the evidence itself has not changed.

I am not convinced that considerations like these really defeat the asymmetry principle, but this question is beyond the scope of this paper. I shall instead argue that the dilemma for Kroedel remains in place even if we grant both of these points. First of all, let me emphasise that the violation of the asymmetry principle is only one part of the first horn of the dilemma. Even if the asymmetry principle turns out to be false, the point remains that it is independently implausible to assume that a body of evidence that indicates that a proposition is extremely likely provides sufficient reasons for revising this belief.

That Kroedel is committed to this assumption (on the first horn of the dilemma) also shows that it does not help him to accept borderline cases as counterexamples to the asymmetry principle. Kroedel is committed to a symmetry of rational belief formation

---

[33] See e.g. Fantl and McGrath (2002).

and revision not only in borderline cases but in *all* cases in which the probability is less than 1. What this means is that we can continuously go back and forth in forming and revising all of our justified beliefs unless they are absolutely certain – and yet remain fully epistemically rational. This goes far beyond accepting that there may be borderline cases in which the asymmetry principle fails to apply. Finally, it also does not help Kroedel to reject the asymmetry principle on grounds of considerations concerning pragmatic encroachment. This is for the simple reason that the relevant phenomena do not call into question that there is an asymmetry between forming and revising beliefs *within one and the same pragmatic context.* And we can simply stipulate that no change of pragmatic context occurs in Lotta's case.

The second objection takes issue with my claim that rejecting the first horn of the dilemma (and thus accepting that there is no sufficient object-given reason for revising any particular lottery belief) commits one to the existence of an epistemic obligation that cannot rationally be satisfied. The proponent of this objection holds that in order to rationally satisfy an obligation not to have the combination of beliefs [$B(l_1)$, and …, and $B(l_{max+1})$], one does not need sufficient object-given reasons for revising any particular lottery belief, one only needs sufficient object-given reasons for revising [$B(l_1)$, or …, or $B(l_{max+1})$]. Since the conjunction of l_1&…&l_{max+1} is, *ex hypothesi,* insufficiently likely, Lotta's evidence may plausibly be taken to provide sufficient object-given reasons for revising [$B(l_1)$, or …, or $B(l_{max+1})$]. And so, the objection continues, Lotta can rationally satisfy her obligation even though she lacks sufficient object-given reason for revising any particular lottery belief.

My response is that in order to revise a particular belief in a rational way, we need sufficient object-given reasons to revise this particular belief. Evidence that shows a conjunction of propositions to be insufficiently likely provides a sufficient basis for rationally revising a belief in this conjunction, but it does not by itself provide a sufficient basis for rationally revising a belief in one of the conjuncts. Note that our evidence quite generally suggests that the conjunction of all of our beliefs is unlikely or even inconsistent – yet this is no reason on the basis of which we can rationally revise one of our particular beliefs.

This response may be challenged by way of an analogy to the practical case. You have promised to give me one out of a set of *n* presents, all of which you like equally well. This is a reason to give me [present_1, or …, or present_n]. Even though it is not a reason to give me one particular present rather than another, you can rationally pick one particular present on the basis of that reason. Why then should it not be rationally possible for us to revise a particular lottery belief on the basis of a reason to revise [$B(l_1)$, or …, or $B(l_{max+1})$]?

I take it that the two cases are not intuitively on a par. Intuitively, one can rationally pick one of the paintings, but one cannot rationally "pick" one of the beliefs. Rational belief revision is not a matter of picking. This intuitive difference is also reflected in the fact that the most natural explanation of why it is possible for us to rationally decide to give away one particular painting does not likewise suggest that it is possible for us to rationally revise one particular lottery belief. The explanation in the practical case is that we have *instrumental reasons* to take means to actions we have reason to perform and that we can rationally base a decision to take a particular means on such instrumental reasons, provided that these reasons are sufficient and that there is no conclusive reason against taking the particular means. If you have sufficient reason to keep your promise, then you also have sufficient reason to take sufficient means to keeping your promise, and since you have no conclusive reason against giving away present_1, and doing so is a sufficient means to keeping your promise, you can rationally base a decision to give away present_1 on this reason.

However, as Jonathan Way first pointed out, such principles of instrumental transmission of reasons do generally not apply to object-given reasons for attitudes.[34] For example, if you have sufficient object-given reasons to revise your belief that God exists, and a necessary or sufficient means for you to doing this is revising your belief that life is meaningful, it does not follow that you have any object-given reason to revise that latter belief – and so it does not follow that you can rationally revise it on the basis of the original object-given reason. More generally speaking, rational belief-formation and -revision is not sensitive to instrumental considerations in the way that rational decision-making is. Therefore, the analogy to the practical case fails. Even if we were to assume

[34] See Way (2010, §4), and esp. Way (2012).

such a thing as an object-given reason to revise [B(l_1), or …, or B(l_{max+1})], it would not follow from this that there is an object-given reason, and thus a rational basis, for revising any of your particular lottery beliefs.

4. Conclusion

Kroedel argues that the lottery paradox can be solved if epistemic justification is identified with epistemic permissibility. He claims that such an account would allow us to reject *justification agglomeration*, and thus to maintain both the probability claim and a version of the conjunction claim, without being committed to the paradoxical conclusion that we have justification for believing that all tickets will lose. I have presented two objections to this solution. First, even if justification amounts to no more than permissibility, there plausibly are epistemic obligations to believe what is very likely to be true if certain background conditions hold, and the lottery paradox can be restated in terms of such obligations. Second, even if there are no epistemic obligations to believe lottery propositions, Kroedel's solution fails because justification agglomerates. Both of these objections show, on independent grounds, that a satisfying solution to the lottery paradox, which accepts the probability claim, needs to reject the conjunction claim in either of the versions distinguished above.[35]

References

Chuard, Philippe, and Nicholas Southwood. 2009. "Epistemic Norms without Voluntary Control." *Noûs* 43 (4): 599–632.

Eder, Anna-Maria Asunta. 2015. "No Match Point for the Permissibility Account." *Erkenntnis* 80 (3): 657–73.

Fantl, Jeremy, and Matthew McGrath. 2002. "Evidence, Pragmatics, and Justification." *The Philosophical Review* 111 (1): 67–94.

Foley, Richard. 1979. "Justified Inconsistent Beliefs." *American Philosophical Quarterly* 16 (4): 247–57.

[35] Draft manuscript, 09 December 2015. Earlier versions of this paper have been presented at the Philosophical Society, Lund University, and the Practical Philosophy/Ethics colloquium at Humboldt-Universität zu Berlin. I would like to thank the participants for helpful discussion, and especially Jan Gertken, Thomas Kroedel, Arturs Logins, and Thomas Schmidt for written comments. Work on this essay has been supported by the Deutsche Forschungsgemeinschaft (DFG Project "Principles of the Deliberative Ought").

———. 1992. "The Epistemology of Belief and the Epistemology of Degrees of Belief." *American Philosophical Quarterly* 29 (2): 111–24.

Harman, Gilbert. 1986. *Change in View. Principles of Reasoning*. Cambridge, MA: MIT Press.

Huber, Franz. 2014. "What Is the Permissibility Solution a Solution of? A Question for Kroedel." *Logos & Episteme* 5 (3): 333–42.

Kiesewetter, Benjamin. 2013. *The Normativity of Rationality*. Ph.D. thesis, Humboldt University of Berlin.

Klein, Peter. 1985. "The Virtues of Inconsistency." *The Monist* 68 (1): 105–35.

Kolodny, Niko. 2005. "Why Be Rational?" *Mind* 114 (455): 509–63.

Kroedel, Thomas. 2012. "The Lottery Paradox, Epistemic Justification and Permissibility." *Analysis* 72 (1): 57–60.

———. 2013a. "The Permissibility Solution to the Lottery Paradox – Reply to Littlejohn." *Logos & Episteme* 4 (1): 103–11.

———. 2013b. "Why Epistemic Permissions Don't Agglomerate – Another Reply to Littlejohn." *Logos & Episteme* 4 (4): 451–55.

Kyburg, Henry E. 1961. *Probability and the Logic of Rational Belief*. Middletown: Wesleyan University Press.

———. 1970. "Conjunctivitis." In *Induction, Acceptance and Rational Belief*, edited by Marshall Swain, 55–82. Dordrecht: D. Reidel.

Littlejohn, Clayton. 2012. "Lotteries, Probabilities, and Permissions." *Logos & Episteme* 3 (3): 509–14.

———. 2013. "Don't Know, Don't Believe: Reply to Kroedel." *Logos & Episteme* 4 (2): 231–38.

Makinson, D. C. 1965. "The Paradox of the Preface." *Analysis* 25 (6): 205–7.

Moran, Richard. 1988. "Making Up Your Mind." *Ratio* 1 (2): 135–51.

Nelkin, Dana K. 2000. "The Lottery Paradox, Knowledge, and Rationality." *The Philosophical Review* 109 (3): 373–409.

Nelson, Mark T. 2010. "We Have No Positive Epistemic Duties." *Mind* 119 (473): 83–102.

Nozick, Robert. 1993. *The Nature of Rationality*. Princeton: Princeton University Press (repr. 1995).

Parfit, Derek. 2001. "Rationality and Reasons." In *Exploring Practical Philosophy: From Action to Values*, edited by Dan Egonsson, Jonas Josefsson, Björn Petersson, and Tonio Rønnow-Rasmussen, 17–39. Aldershot: Ashgate.

———. 2011. *On What Matters*. Vol. 1. Oxford: Oxford University Press.

Piller, Christian. 2006. "Content-Related and Attitude-Related Reasons for Preferences." *Royal Institute of Philosophy Supplements* 81 (59): 155–81.

Ryan, Sharon. 1996. "The Epistemic Virtues of Consistency." *Synthese* 109 (2): 121–41.

Schroeder, Mark. 2012. "The Ubiquity of State-Given Reasons." *Ethics* 122 (3): 457–88.

Shah, Nishi, and Matthew Silverstein. 2013. "Reasoning in Stages." *Ethics* 124 (1): 101–13.

Skorupski, John. 2010. *The Domain of Reasons*. Oxford: Oxford University Press.

Way, Jonathan. 2010. "Defending the Wide-Scope Approach to Instrumental Reason." *Philosophical Studies* 147: 213–33.

———. 2012. "Transmission and the Wrong Kind of Reason." *Ethics* 122 (3): 489–515.